DIVERGÊNCIA E TEOREMA DA DIVERGÊNCIA
DIVERGENCE AND DIVERGENCE THEOREM

1) Mostrar que o divergente da densidade de fluxo eléctrico gerado por uma carga pontual é nulo. Show that the divergent of the electric flux density generated by a point charge is zero.

$$\vec{E} = \frac{Q}{4\pi\varepsilon R^2}\vec{a}_R \qquad \vec{D} = \varepsilon\vec{E} \qquad div(\vec{A}) = \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\operatorname{sen}\theta}\frac{\partial}{\partial\theta}(A_\theta \operatorname{sen}\theta) + \frac{1}{r\operatorname{sen}\theta}\frac{\partial A_\phi}{\partial\phi}$$

Resolução Resolution

$$\vec{D} = \frac{Q}{4\pi\varepsilon r^2}\vec{a}_R\,\varepsilon$$

$$\vec{D} = \frac{Q}{4\pi r^2}\vec{a}_R + 0\vec{a}_\theta + 0\vec{a}\phi$$

coordenadas esfericas spherical coordinates

$$\nabla \cdot \vec{D} = \frac{1}{r^2}\frac{\partial(r^2 D_r)}{\partial r} + \frac{1}{r\operatorname{sen}\theta}\frac{\partial}{\partial\theta}(D_\theta \operatorname{sen}\theta) + \frac{1}{r\operatorname{sen}\theta}\frac{\partial D_\phi}{\partial\phi}$$

$$\nabla \cdot \vec{D} = \frac{1}{r^2}\frac{\partial\left(r^2 \frac{Q}{4\pi r^2}\right)}{\partial r} + \frac{1}{r\operatorname{sen}\theta}\frac{\partial}{\partial\theta}(0\operatorname{sen}\theta) + \frac{1}{r\operatorname{sen}\theta}\frac{\partial 0}{\partial\phi}$$

$$\nabla \cdot \vec{D} = \nabla \cdot \frac{Q}{4\pi r^2}\vec{a}_R = 0 + 0 + 0 \qquad \nabla \cdot \vec{D} = 0$$

Resposta Answer

$$\nabla \cdot \vec{D} = 0$$

2) Mostrar que o divergente da densidade de fluxo eléctrico gerado por uma linha recta infinita uniformemente carregada é nulo. Show that the divergent of the electric flux density generated by a uniformly charged infinite straight line is zero.

$$\vec{E} = \frac{\rho_l}{2\pi\varepsilon r}\vec{a}_R \qquad \vec{D} = \varepsilon\vec{E} \qquad div(\vec{A}) = \nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial\phi} + \frac{\partial A_z}{\partial z}$$

Resolução Resolution

$$\vec{D} = \frac{\rho_l}{2\pi\varepsilon r}\vec{a}_R\varepsilon$$

$$\vec{D} = \frac{\rho_l}{2\pi r}\vec{a}_R + 0\vec{a}_\phi + 0\vec{a}_Z$$ coordenadas cilíndricas cylindrical coordinates

$$\nabla\cdot\vec{D} = \frac{1}{r}\frac{\partial(r D_r)}{\partial r} + \frac{1}{r}\frac{\partial D_\phi}{\partial\phi} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\vec{D} = \frac{1}{r}\frac{\partial\left(r\frac{\rho_l}{2\pi r}\right)}{\partial r} + \frac{1}{r}\frac{\partial 0}{\partial\phi} + \frac{\partial 0}{\partial z}$$

$$\nabla\cdot\vec{D} = \nabla\cdot\frac{\rho_l}{2\pi r}\vec{a}_R = 0+0+0 \qquad \nabla\cdot\vec{D} = 0$$

Resposta Answer

$$\nabla\cdot\vec{D} = 0$$

3) Mostrar que o divergente da densidade de fluxo eléctrico gerado por um plano uniformemente carregado é nulo. Show that the divergent of the electric flux density generated by uniformly charged plane is zero.

$$\vec{E} = \frac{\rho_s}{2\varepsilon}\vec{a}_R \qquad \vec{D} = \varepsilon\vec{E} \qquad div(\vec{A}) = \nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Resolução Resolution

$$\vec{D} = \frac{\rho_s}{2\varepsilon}\vec{a}_R\,\varepsilon$$

$$\vec{D} = \frac{\rho_s}{2}\vec{a}_X + 0\vec{a}_Y + 0\vec{a}_Z$$ coordenadas cartesianas cartesian coordinates

$$\nabla\cdot\vec{D} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}$$

$$\nabla\cdot\vec{\mathrm{D}}=\frac{\partial\frac{\rho_s}{2}}{\partial\mathrm{x}}+\frac{\partial 0}{\partial\mathrm{y}}+\frac{\partial 0}{\partial\mathrm{z}}$$

$$\nabla\cdot\vec{\mathrm{D}}=\nabla\cdot\frac{\rho_s}{2}\vec{a}_X=0+0+0 \qquad \nabla\cdot\vec{\mathrm{D}}=0$$

Resposta Answer

$$\nabla\cdot\vec{\mathrm{D}}=0$$

4) Considere uma esfera isolante electrizada no vazio. Como se sabe, o campo eléctrico por ela criado à distância R da sua superfície é o dado pelas expressões ao lado. (adapte devidamente a expressão ao referencial) Consider an insulated sphere in the vacuum with uniformly electrical charge distribution. As is known, the electric field created by it at the distance R from the surface is given by the expressions on the left. (suitably adapt to the expression reference)

$$\vec{\mathrm{E}}=\frac{K^3\rho_v}{3\varepsilon(K+r)^2}\vec{a}_R;r\geq 0$$

$$\vec{\mathrm{E}}=\frac{(K+r)\rho_v}{3\varepsilon}\vec{a}_R;r<0$$

a) Determine analiticamente o divergente da densidade de fluxo eléctrico gerado pela carga armazenada na esfera dentro e fora dela. Establish analytically the divergence of the electric flux density generated by the electrical stored charge on the ball inside and outside it.

$$\vec{\mathrm{D}}=\varepsilon\vec{\mathrm{E}}$$

$$div(\vec{\mathrm{A}})=\nabla\cdot\vec{\mathrm{A}}=\frac{1}{\mathrm{r}^2}\frac{\partial(\mathrm{r}^2\mathrm{A}_r)}{\partial\mathrm{r}}+\frac{1}{\mathrm{rsen}\,\theta}\frac{\partial}{\partial\theta}(\mathrm{A}_\theta\mathrm{sen}\,\theta)+\frac{1}{\mathrm{rsen}\,\theta}\frac{\partial\mathrm{A}_\phi}{\partial\phi}$$ coordenadas esféricas spherical coordinates

Resolução Resolution

$$\vec{\mathrm{D}}=\frac{K^3\rho_v}{3\varepsilon r^2}\varepsilon\vec{a}_R \quad ;r\geq K \qquad \vec{\mathrm{D}}=\frac{K^3\rho_v}{3r^2}\vec{a}_R+0\vec{a}_\theta+0\vec{a}_\phi \quad ;r\geq K$$

$$\vec{\mathrm{D}}=\frac{r\rho_v}{3\varepsilon}\varepsilon\vec{a}_R \quad ;r<K \qquad \vec{\mathrm{D}}=\frac{r\rho_v}{3}\vec{a}_R+0\vec{a}_\theta+0\vec{a}_\phi \quad ;r<K$$

$$\nabla\cdot\vec{\mathrm{D}}=\frac{1}{\mathrm{r}^2}\frac{\partial\left(r^2\frac{K^3\rho_v}{3r^2}\right)}{\partial\mathrm{r}}+\frac{1}{\mathrm{rsen}\,\theta}\frac{\partial}{\partial\theta}(0\,\mathrm{sen}\,\theta)+\frac{1}{\mathrm{rsen}\,\theta}\frac{\partial 0}{\partial\phi} \quad ;r\geq K$$

$$\nabla\cdot\vec{\mathrm{D}}=\frac{1}{\mathrm{r}^2}\frac{\partial\left(r^2\frac{r\rho_v}{3}\right)}{\partial\mathrm{r}}+\frac{1}{\mathrm{rsen}\,\theta}\frac{\partial}{\partial\theta}(0\,\mathrm{sen}\,\theta)+\frac{1}{\mathrm{rsen}\,\theta}\frac{\partial 0}{\partial\phi} \quad ;r<K$$

$$\nabla\cdot\vec{\mathrm{D}}=\frac{K^3\rho_v}{3\mathrm{r}^2}\frac{\partial 1}{\partial \mathrm{r}}+0+0 \quad ;r\geq K$$
$$\nabla\cdot\vec{\mathrm{D}}=\frac{\rho_v}{3\mathrm{r}^2}\frac{\partial r^3}{\partial \mathrm{r}}+0+0 \quad ;r< K$$

$$\nabla\cdot\vec{\mathrm{D}}=\frac{K^3\rho_v}{3\mathrm{r}^2}0+0+0 \quad ;r\geq K$$
$$\nabla\cdot\vec{\mathrm{D}}=\rho_v+0+0 \quad ;r< K$$

$$\nabla\cdot\vec{\mathrm{D}}=0 \quad ;r\geq K$$
$$\nabla\cdot\vec{\mathrm{D}}=\rho_v \quad ;r< K$$

Resposta Answer

$\nabla\cdot\vec{\mathrm{D}}=0 \quad ;r\geq K$ fora da esfera outside of the sphere

$\nabla\cdot\vec{\mathrm{D}}=\rho_v \quad ;r< K$ (C/m^3) interior da esfera inside the sphere

b) Recorrendo do teorema da divergência, encontre a expressão da carga total armazenada na esfera. Confronte este resultado com o obtido anteriormente pela aplicação da lei de Gauss. Using the divergence theorem, find the expression of the total charge stored in the sphere. Confront this result with that obtained previously by application of Gauss' law.

$\int_{\mathrm{v}}\left(\nabla\cdot\vec{\mathrm{D}}\right)d\mathrm{v}=\mathrm{Q}_{\mathrm{int}}$ da questão anterior from the last question $\nabla\cdot\vec{\mathrm{D}}=\rho_v \quad ;r< K$

$$dv=r^2\sin(\theta)\,dr\,d\theta\,d\phi$$

$$\int_0^{2\pi}\int_0^{\pi}\int_0^{K}\rho_v r^2\sin(\theta)\,dr\,d\theta\,d\phi=\mathrm{Q}_{\mathrm{int}}$$

$$\frac{\rho_v}{3}\int_0^{2\pi}\int_0^{\pi}\left[r^3\right]_0^K\sin(\theta)d\theta\,d\phi=\mathrm{Q}_{\mathrm{int}}$$

$$\frac{\rho_v}{3}K^3\int_0^{2\pi}\int_0^{\pi}\sin(\theta)d\theta\,d\phi=\mathrm{Q}_{\mathrm{int}}$$

$$-\frac{\rho_v}{3}K^3\int_0^{2\pi}\left[\cos(\theta)\right]_0^{\pi}d\phi=\mathrm{Q}_{\mathrm{int}}$$

$$-\frac{\rho_v}{3}K^3\int_0^{2\pi}(-1-1)d\phi=\mathrm{Q}_{\mathrm{int}}$$

$$2\frac{\rho_v}{3}K^3\int_0^{2\pi}d\phi=\mathrm{Q}_{\mathrm{int}}$$

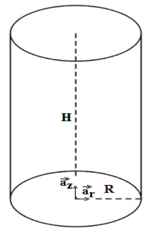
$$\frac{4}{3}\pi K^3\rho_v=\mathrm{Q}_{\mathrm{int}}$$

Resposta Answer

$\frac{4}{3}\pi\mathrm{K}^3\rho_{\mathrm{v}}=\mathrm{Q}_{\mathrm{int}}$ a mesma encontrada anteriormente the same found previously

5) Considere um cilindro electrizado para o qual a equação da densidade de fluxo eléctrico no seu interior dada pela equação ao lado, em coordenadas cilíndricas. (adapte devidamente a expressão ao referencial) Assume an cylinder for which the inner electric flux density is given by the equation in the left, in cylindrical coordinates. (suitably adapt to the expression reference)

$$\vec{\mathrm{D}}=30e^{-r}\vec{\mathrm{a}}_r-2z\vec{\mathrm{a}}_z \quad (\mathrm{C/m^2})$$



a) Determine analiticamente o divergente da densidade de fluxo eléctrico no interior do cilindro e fora dele. Analytically find the divergent of the electric flux density inside and outside of the cylinder.

$$div(\vec{A}) = \nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \qquad \vec{D} = 30e^{-r}\vec{a}_r - 2z\vec{a}_z \quad (C/m^2)$$

$$\nabla \cdot \vec{D} = \frac{1}{r}\frac{\partial\left(r\,30e^{-r}\right)}{\partial r} + \frac{1}{r}\frac{\partial 0}{\partial \phi} + \frac{\partial(-2z)}{\partial z} \qquad \nabla \cdot \vec{D} = \frac{30}{r}\frac{\partial\left(re^{-r}\right)}{\partial r} + 0 - 2$$

$$\nabla \cdot \vec{D} = \frac{30}{r}\left(e^{-r} + r\,e^{-r}(-1)\right) + 0 - 2 \qquad \nabla \cdot \vec{D} = \frac{30\,e^{-r}}{r}(1-r) - 2$$

Resposta Answer

$\nabla \cdot \vec{D} = \frac{30\,e^{-r}}{r}(1-r) - 2$ (C/m^3) no interior do cilindro inside the cylinder

b) Calcule os dois lados do teorema da divergência e encontre a expressão da carga total armazenada no cilindro. O cilindro tem altura H e raio R de 10cm e 1cm, respectivamente. Calculate the two sides of the divergence theorem and find the expression of the total charge stored in the cylinder. The cylinder has a height H and the radius R of 10 cm and 1 cm, respectively.

$$\oint \vec{D}\cdot d\vec{S} = \int_v \left(\nabla \cdot \vec{D}\right)dv = Q_{int} \qquad \vec{D} = 30e^{-r}\vec{a}_r - 2z\vec{a}_z \quad (C/m^2) \qquad \nabla \cdot \vec{D} = \frac{30\,e^{-r}}{r}(1-r) - 2$$

$$ds_{r\phi} = rdrd\phi \qquad ds_{z\phi} = rdzd\phi \qquad dv = r\,dr\,d\phi\,dz$$

$$\int_0^{2\pi}\int_0^R (30\,e^{-r}\vec{a}_R - 2\times 0\times \vec{a}_z)\cdot rdrd\phi(-\vec{a}_Z) + \int_0^{2\pi}\int_0^H (30\,e^{-R}\vec{a}_R - 2z\vec{a}_z)\cdot Rdzd\phi\,\vec{a}_R + \int_0^{2\pi}\int_0^R (30\,e^{-r}\vec{a}_R - 2H\,\vec{a}_z)\cdot rdrd\phi\,\vec{a}_Z = Q_{int}$$

$$\int_0^H\int_0^{2\pi}\int_0^R \left(30(1-r)\,e^{-r} - 2r\right)dr\,d\phi\,dz = Q_{int}$$

$$30R\,e^{-R}\int_0^{2\pi}\int_0^H dzd\phi - 2H\int_0^{2\pi}\int_0^R rdrd\phi = Q_{int} \qquad 2\pi H\,30R\,e^{-R} - 2\pi HR^2 = Q_{int} \qquad 2\pi HR(30\,e^{-R} - R) = Q_{int}$$

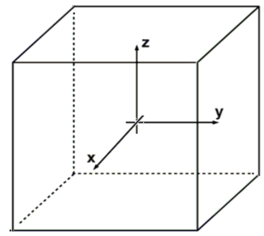
$$\int_0^H\int_0^{2\pi}\int_0^R \left(30(1-r)\,e^{-r} - 2r\right)dr\,d\phi\,dz = Q_{int} \qquad 2\pi H\,30R\,e^{-R} - 2\pi HR^2 = Q_{int} \qquad 2\pi HR(30\,e^{-R} - R) = Q_{int}$$

Resposta Answer

$2\pi HR(30\,e^{-R} - R) = Q_{int}$ as duas formas dão o mesmo resultado both ways yield the same result

6) Considere um zona cúbica com 10 cm de lado centrada na origem dum referencial cartesiano ortonormal. A densidade de fluxo eléctrico na região interior do cubo é dada pela expressão ao lado. (adapte devidamente a expressão ao referencial) Consider a cubic area with 10cm side centered at the origin of a Cartesian orthonormal referential. The electric flux density in the inner region of the cube is given by the equation at the left. (suitably adapt the expression to the reference)

$$\vec{D} = \left[x^2\vec{a}_x + yz\vec{a}_y + xy\vec{a}_z\right] \quad (\mathrm{C/m^2})$$



a) Determine analiticamente o divergente da densidade de fluxo eléctrico no interior do cubo. Find analytically the divergent of the electric flux density inside the cube.

$$\vec{D} = \left[x^2\vec{a}_x + yz\vec{a}_y + xy\vec{a}_z\right] \quad (\mathrm{C/m^2}) \qquad div(\vec{A}) = \nabla\cdot\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\cdot\vec{D} = \frac{\partial x^2}{\partial x} + \frac{\partial yz}{\partial y} + \frac{\partial xy}{\partial z} \qquad \nabla\cdot\vec{D} = 2x + z + 0$$

Resposta Answer

$$\nabla\cdot\vec{D} = 2x + z \quad (\mathrm{C/m^3})$$

b) Analise os dois lados do teorema da divergência e encontre a expressão da carga total armazenada no cubo. Examine both sides of the divergence theorem and find the expression of the total charge stored in the cube.

$$\oint \vec{D}\cdot d\vec{S} = \int_v \left(\nabla\cdot\vec{D}\right)dv = Q_{int} \qquad \vec{D} = \left[x^2\vec{a}_x + yz\vec{a}_y + xy\vec{a}_z\right] \quad (\mathrm{C/m^2})$$

$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(x^2\vec{a}_x + y\frac{L}{2}\vec{a}_y + xy\vec{a}_z)\cdot dxdy\vec{a}_z + \int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(x^2\vec{a}_x + y\frac{-L}{2}\vec{a}_y + xy\vec{a}_z)\cdot dxdy(-\vec{a}_z) +$$
$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(x^2\vec{a}_x + \frac{L}{2}z\vec{a}_y + x\frac{L}{2}\vec{a}_z)\cdot dxdz\vec{a}_y + \int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(x^2\vec{a}_x - \frac{L}{2}z\vec{a}_y - x\frac{L}{2}\vec{a}_z)\cdot dxdz(-\vec{a}_y) +$$
$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\left(\left(\frac{L}{2}\right)^2\vec{a}_x + yz\vec{a}_y + \frac{L}{2}y\vec{a}_z\right)\cdot dydz\vec{a}_x + \int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\left[\left(-\frac{L}{2}\right)^2\vec{a}_x + yz\vec{a}_y - \frac{L}{2}y\vec{a}_z\right]\cdot dydz(-\vec{a}_x) = Q_{int}$$

$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}(2x + z)dxdydz = Q_{int}$$

$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \mathrm{x}\mathrm{y}dxdy - \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \mathrm{x}\mathrm{y}dxdy + \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \frac{L}{2} zdxdz + \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \frac{L}{2} zdxdz + \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \left(\frac{L}{2}\right)^2 dydz - \int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \left(\frac{L}{2}\right)^2 dydz = \mathrm{Q}_{\mathrm{int}}$$

$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \left[\mathrm{x}^2 + \mathrm{z}\,\mathrm{x}\right]_{-L/2}^{L/2} dydz = \mathrm{Q}_{\mathrm{int}}$$

$$L\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} zdxdz = \mathrm{Q}_{\mathrm{int}} \qquad L^2\int_{-L/2}^{L/2} zdz = \mathrm{Q}_{\mathrm{int}} \qquad \frac{L^2}{2}\left[z^2\right]_{-L/2}^{L/2} = \mathrm{Q}_{\mathrm{int}} \qquad 0 = \mathrm{Q}_{\mathrm{int}}$$

$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2} \left[\mathrm{x}^2 + \mathrm{z}\,\mathrm{x}\right]_{-L/2}^{L/2} dydz = \mathrm{Q}_{\mathrm{int}} \qquad \frac{L^2}{2}\left[z^2\right]_{-L/2}^{L/2} = \mathrm{Q}_{\mathrm{int}} \qquad 0 = \mathrm{Q}_{\mathrm{int}} \qquad 0 = \mathrm{Q}_{\mathrm{int}}$$

Resposta Answer

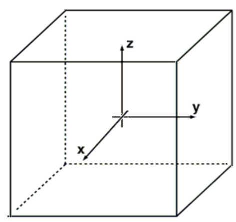
$$0 = \mathrm{Q}_{\mathrm{int}}$$

c) Determine a carga total armazenada no cubo. Find the total electrical charge stored inside the cube.

$$0 = \mathrm{Q}_{\mathrm{int}}$$

7) Considere uma zona cúbica de aresta 1 cm do espaço definida num referencial cartesiano ortonormal. A densidade de fluxo eléctrico na região é dada pela expressão ao lado. (adapte devidamente a expressão ao referencial) Consider a cubic region of space with edge of 1 cm defined in a cartesian orthonormal frame. The electric flux density in the region is given by the equation at left. (suitably adapt the expression to the reference)

$$\vec{\mathrm{D}} = 5\,\mathrm{x}^2 \mathrm{sen}\left(\frac{\pi}{2}\mathrm{x}\right)\vec{\mathrm{a}}_{\mathrm{x}} \qquad (\mathrm{C/m^2})$$



a) Determine analiticamente o divergente da densidade de fluxo eléctrico na região. Find analytically the divergent of the electric flux density in the region.

$$\vec{\mathrm{D}} = 5\,\mathrm{x}^2 \mathrm{sen}\left(\frac{\pi}{2}\mathrm{x}\right)\vec{\mathrm{a}}_{\mathrm{x}} \qquad (\mathrm{C/m^2})$$

$$div(\vec{\mathrm{A}}) = \nabla\cdot\vec{\mathrm{A}} = \frac{\partial \mathrm{A_x}}{\partial \mathrm{x}} + \frac{\partial \mathrm{A_y}}{\partial \mathrm{y}} + \frac{\partial \mathrm{A_z}}{\partial \mathrm{z}}$$

$$\nabla\cdot\vec{\mathrm{D}} = \frac{\partial 5x^2\,\mathrm{sen}\left(\frac{\pi}{2}x\right)}{\partial x} + \frac{\partial 0}{\partial \mathrm{y}} + \frac{\partial 0}{\partial \mathrm{z}}$$

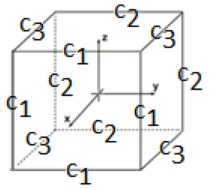
$$\nabla\cdot\vec{\mathrm{D}} = 10x\,\mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}x^2\cos\left(\frac{\pi}{2}x\right)$$

Resposta Answer

$$\nabla\cdot\vec{\mathrm{D}} = 10\,\mathrm{x}\,\mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}\mathrm{x}^2\cos\left(\frac{\pi}{2}x\right) \qquad (\mathrm{C/m^3})$$

b) Determine a densidade de carga eléctrica nos vértices do cubo. Find the electrical charge density in the corners of the cube.

$$\nabla \cdot \vec{D} = 10\,x\ \mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}x^2 \cos\left(\frac{\pi}{2}x\right)$$



C_1->x=0.5 cm
C_2-> x=-0.5 cm
C_3-> -0.5 cm < x < 0.5 cm

$$C_1 = 10\frac{1}{2}\ \mathrm{sen}\left(\frac{\pi}{2}\frac{1}{2}\right) + \frac{5\pi}{2}\left(\frac{1}{2}\right)^2 \cos\left(\frac{\pi}{2}\frac{1}{2}\right) \qquad C_1 = 5\frac{\sqrt{2}}{2}\left(1+\frac{\pi}{8}\right)$$

$$C_2 = 10\left(-\frac{1}{2}\right)\mathrm{sen}\left(-\frac{1}{2}\frac{\pi}{2}\right) + \frac{5\pi}{2}\left(\frac{1}{2}\right)^2 \cos\left(-\frac{\pi}{2}\frac{1}{2}\right) \qquad C_2 = 5\frac{\sqrt{2}}{2}\left(1+\frac{\pi}{8}\right)$$

$$C_3 = 10x\ \mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}(x)^2 \cos\left(\frac{\pi}{2}x\right) \qquad C_3 = 10x\ \mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}(x)^2 \cos\left(\frac{\pi}{2}x\right)$$

Resposta Answer

$$C_1 = C_2 = 5\frac{\sqrt{2}}{2}\left(1+\frac{\pi}{8}\right) \quad (\mathrm{C/m^3}) \qquad C_3 = 10x\ \mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}(x)^2 \cos\left(\frac{\pi}{2}x\right) \quad (\mathrm{C/m^3})$$

c) Determine a expressão da carga interior ao cubo. Find the expression of the electrical charge inner to the cube.

$$\oint \vec{D}\cdot d\vec{S} = \int_v \left(\nabla \cdot \vec{D}\right) dv = Q_{\mathrm{int}} \qquad \nabla \cdot \vec{D} = 10\,x\ \mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}x^2 \cos\left(\frac{\pi}{2}x\right)$$

$$\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\int_{-L/2}^{L/2}\left(10x\ \mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}x^2\cos\left(\frac{\pi}{2}x\right)\right)dxdydz = Q_{\mathrm{int}}$$

$$L^2\int_{-L/2}^{L/2}\left(10x\ \mathrm{sen}\left(\frac{\pi}{2}x\right) + \frac{5\pi}{2}x^2\cos\left(\frac{\pi}{2}x\right)\right)dx = Q_{\mathrm{int}} \qquad L^2\left[5x^2\sin\left(\frac{\pi}{2}x\right)\right]_{-L/2}^{L/2} = Q_{\mathrm{int}}$$

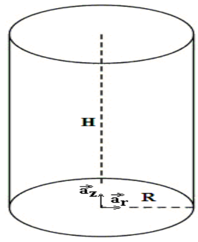
$$5L^2\left[\frac{L^2}{4}\sin\left(\frac{\pi L}{4}\right) - \frac{L^2}{4}\sin\left(-\frac{\pi L}{4}\right)\right] = Q_{\mathrm{int}} \qquad L^4\frac{5}{2}\sin\left(\frac{\pi L}{4}\right) = Q_{\mathrm{int}}$$

Resposta Answer

$$L^4\frac{5}{2}\sin\left(\frac{\pi L}{4}\right) = Q_{\mathrm{int}}$$

8) Considere uma zona do espaço definida num referencial cilíndrico. A densidade de fluxo eléctrico na região é dada pela expressão ao lado. (adapte devidamente a expressão ao referencial) Consider a region of space defined in a cylindrical reference system. The electric flux density in the region is given by the equation at left. (suitably adapted the expression to the reference)

$$\vec{D} = r\,\text{sen}(\phi)\vec{a}_r + r^2\cos(\phi)\vec{a}_\phi + 2re^{-5z}\,\vec{a}_z$$



a) Determine analiticamente o divergente da densidade de fluxo eléctrico na região. Find analytically the divergent of the electric flux density in the region.

$$div(\vec{A}) = \nabla \cdot \vec{A} = \frac{1}{r}\frac{\partial(rA_r)}{\partial r} + \frac{1}{r}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z} \qquad \vec{D} = r\,\text{sen}(\phi)\vec{a}_r + r^2\cos(\phi)\vec{a}_\phi + 2re^{-5z}\,\vec{a}_z$$

$$\nabla \cdot \vec{D} = \frac{\text{sen}(\phi)}{r}\frac{\partial(r^2)}{\partial r} + r\frac{\partial\cos(\phi)}{\partial \phi} + 2r\frac{\partial e^{-5z}}{\partial z} \qquad \nabla \cdot \vec{D} = 2\,\text{sen}(\phi) - r\sin(\phi) - 10re^{-5z}$$

b) Determine a densidade de carga no ponto (1/2, π /2, 0). Find the electrical charge density at the point (1/2, π/2, 0).

$$(r,\phi,z) = (1/2, \pi/2, 0) \qquad \rho = \nabla \cdot \vec{D} = 2\,\text{sen}(\phi) - r\sin(\phi) - 10re^{-5z}$$

$$\rho = 2\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - 10\frac{1}{2}e^{-5*0} \qquad \rho = 2\sin\left(\frac{\pi}{2}\right) - \frac{1}{2}\sin\left(\frac{\pi}{2}\right) - 10\frac{1}{2}e^{-5*0} \qquad \rho = \frac{7}{2}$$

Resposta Answer

$$\rho = -\frac{7}{2} \qquad (\text{C/m}^3)$$

9) Considere uma zona do espaço definida num referencial cartesiano. A densidade de fluxo eléctrico na região -1<z<1 é dada pela expressão (1) e pala expressão (2) fora dela. (adapte devidamente a expressão ao referencial) Assume a region of the space defined in a cartesian reference frame. The electric flux density inside the region -1 <z <1 is given by the expression (1) and by the expression (2) outside. (suitably adapt the expression to the reference frame)

$$(1) \quad \vec{D} = \rho_0 z\,\vec{a}_z \quad (\text{C/m}^2); \quad -1 < z < 1$$

$$(2) \quad \vec{D} = \rho_0 \frac{z}{|z|}\vec{a}_z \quad (\text{C/m}^2); \quad z < -1 \cup z > 1$$

a) Determine analiticamente o divergente da densidade de fluxo eléctrico nas regiões (1) e (2). Find analytically the divergent of the electric flux density in regions (1) and (2).

$$div(\vec{A}) = \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

(1) $\vec{D} = \rho_0 z \vec{a}_z \quad (C/m^2); \quad -1 < z < 1$

(2) $\vec{D} = \rho_0 \frac{z}{|z|} \vec{a}_z \quad (C/m^2); \quad z < -1 \cup z > 1$

(1) $\nabla \cdot \vec{D} = \partial \frac{\rho_0 z}{\partial z} \qquad -1 < z < 1$

(2) $\nabla \cdot \vec{D} = \partial \frac{\rho_0 \frac{z}{|z|}}{\partial z} \qquad z < -1 \cup z > 1$

(1) $\nabla \cdot \vec{D} = \rho_0 \qquad -1 < z < 1$

(2) $\nabla \cdot \vec{D} = 0 \qquad z < -1 \cup z > 1$

Resposta Answer

(1) $\nabla \cdot \vec{D} = \rho_0 \qquad -1 < z < 1$

(2) $\nabla \cdot \vec{D} = 0 \qquad z < -1 \cup z > 1$

b) Determine a densidade de carga nas duas regiões. Find the electrical charge density in the two regions.

Resposta Answer

Calculado anteriormente Previously calculated

(1) $\nabla \cdot \vec{D} = \rho_0 \qquad -1 < z < 1$

(2) $\nabla \cdot \vec{D} = 0 \qquad z < -1 \cup z > 1$